Answers for class prep quiz on section 4.2, Stewart's Calculus (8th ed.)

1. **Answer:** (d). Neither (a) nor (b) is always true; we cannot guarantee that $f'(x) < 0$ everywhere between 2 and 8, nor can we guarantee that the instantaneous rate of change is equal to the average rate of change at any particular value of $x$ in $[2, 8]$. (c) is true, but that is the definition of average rate of change, not the Mean Value Theorem.

2. **Answer:** (a). The average rate of change of $f(x)$ on $[-1, 3]$ is

$$\frac{f(3) - f(-1)}{3 - (-1)} = \frac{28 - 0}{4} = 7,$$

so the equation to solve to find the "Mean Value Theorem" $x$ values is $f'(x) = 7$, or $3x^2 - 6x + 6 = 7$. (Incidentally, there are two such $x$ values in $[-1, 3]$: $x = 1 + \frac{2\sqrt{3}}{3}$ and $x = 1 - \frac{2\sqrt{3}}{3}$.)

3. **Answer:** (b). Statement II is always true because if $f'(x)$ is always positive, then for $a \le a_1 < b_1 \le b$, since

$$\frac{f(b_1) - f(a_1)}{b_1 - a_1} = f'(c) > 0$$

for some $c$ between $a_1$ and $b_1$, we see that

$$f(b_1) - f(a_1) = (b_1 - a_1)f'(c) > 0.$$

Statement I need not always be true; for example, $f(x) = x^3$ is strictly increasing on $[-1, 1]$, but $f'(0) = 0$.

4. **Answer:** (a). Statement (d) is perhaps the best way to think about Rolle's Theorem vs. the Mean Value Theorem, as the point is that for any function $f(x)$ to which they both apply, Rolle's Theorem and the Mean Value Theorem give you exactly the same information. Of course, since the main application of Rolle's Theorem is to prove the Mean Value Theorem (see statement (b)), for our purposes, we can now just forget about Rolle's Theorem and think only about the Mean Value Theorem.